Course Description: Math 110:772

**Topics in (applied) homotopy theory:** topological insulators in condensed matter physics

The 2016 Nobel Prize in Physics

https : //www.quantamagazine.org/20161004 − nobel − prize − awarded − for − quantum − topology

recognized the importance of mathematical ideas (the theory of topological insulators) in condensed matter physics: see for example

https : //www.quantamagazine.org/20150702 − paradoxical − crystal − baffles − physicists

or click on the 'advanced information' link at

https : //www.nobelprize.org/nobel$_p$rizes/physics/laureates/2016/advanced.html

for an account some of the work behind this.

In a remarkable example of synchronicity (or by just plain being ahead of the curve) Dan Freed and Mike Hopkins recently posted a beautiful account [Reflection positivity and invertible topological phases,

https : //arxiv.org/abs/1604.06527]

on the mathematics underlying these questions. The purpose of this topics course is to go through as much of this paper as possible.

One of the virtues of F&H's paper is its concern with fundamental issues, such as the relation between the mathematically elegant but unrealistic 'Euclidean' approach to quantum field theory, and the questions encountered in the Lorentzian geometry of our (supposedly) real world. Section §2 of their paper for an introduction to this topic (called

https : //en.wikipedia.org/wiki/Wick_rotation

by physicists). Their work is also a kind of showcase for the power of modern techniques in homotopy theory (eg the Madsen-Tillmann-Galatius-Weiss cobordism theory) in subtle questions of physics.

**Note**, that these mathematical techniques evolved out of questions motivated by string theory; but that the 'quantum topology' involved in these condensed matter questions is completely independent of string theory.

[MW 1:30 - 2:45; for further information contact jack@math.jhu.edu]