MATH 756 -- Topics in Algebra: Homological algebra in non-abelian settings.

Fall 2017

Instructor: Prof. Katia Consani

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Class Times: T-TH, 11:00-12:15 pm.
Room: 410B.

References: The suggested textbook for this course is:

M. Grandis, *Homological Algebra in strongly non-abelian settings,* World Scientific.

The following references may be useful for complementary reading:

A. Grothendieck, Sur quelques points d'algebre homologique.

B. Mitchell, Theory of Categories.

S. MacLane, Homology.

Outline of the course: The core of homological algebra is the study of exact sequences and their preservation properties by functors. It was established in categories of modules by Cartan and Eilenberg and extended to abelian categories by Grothendieck, with formal advantages (e.g. duality) and a concrete extension of its domain to sheaves, Serre's quotients etc. In this course I will present the essence of the recent approach, due to M. Grandis, to a development of homological algebra for non abelian categories in the case of semiexact and homological categories. The initial goal of this research was to set up a categorical frame, extending abelian categories (and also exact categories,in the sense of Puppe-Mitchell), that would be sufficiently general to include a codomain for homotopy theory yet sufficiently rich to have good homological properties. The concrete problem is to embed groups, pointed sets, actions of groups (or whatever is needed for the codomain of homotopy theory) in some category with a reasonable notion of exactness. The formal problem is to evince a weak notion of exactness, allowing one to study satellites and spectral sequences. Finally, the central difficulty is to put these things together.

As an application of this theory I will outline the recent development obtained by working with the category of modules over the idempotent Boolean semifield B={0,1}.

Prerequisites: Abstract algebra: including groups, rings and ideals, fields and Galois theory.

Special Notice: This course is listed as a graduate-level topic course and will be developed as such even in the presence of undergraduate students or graduate students in other subjects i.e. without a full undergraduate math major. That means I will expect a level of scholarly and mathematical maturity appropriate to a graduate student in mathematics. In particular, material may go somewhat quickly and students will also be expected to pick up some of it on their own. I warmly suggest ALL STUDENTS ENROLLED to take notes.

Grading: The final grade will be determined from presence and participation at class time.

Important Note: First class Tuesday 09/05 in my office.