In lieu of a syllabus for 110.439 (differential geometry)

Differential geometry can viewed as a kind of continuation of vector calculus (ie Calc III), to infinity and beyond – all the way down the throat of the Schwarzschild (black hole) solution of Einstein’s equations for general relativity.

The table of contents below refers to a set of notes for the course, which is too big to post here. If you’re interested, please contact me at jack@math.jhu.edu and I’ll send you a copy, which will give you a better sense of the material covered in the course.

§I Local vector calculus
1 Some Calc III and linear algebra, p 3 - 5
2 Curves and trajectories: Newton ⇒ Kepler, the tumbling asteroid, p 6 - 9
3 The moving (Frenet) frame, p 10 - 14 ; 15 - 16
4 Parametrized surfaces in $\mathbb{R}^3$; the implicit function theorem, p 18 - 20
5 Curves on surfaces; arc length and area; the 1st fundamental form, p 21 - 23

§II Coordinate systems and the tangent bundle
1 Change of coordinates; the tangent space (see also p 13) and the chain rule, p 23 - 26
2 Digression on higher dimensions: the Schwarzschild metric, p 27 - 29
3 The 2nd fundamental form; ex’s: the helicoid and torus, p 29 - 32
4 Eigenvectors and principal directions; ex: the catenoid, p 33 - 36

§III Covariant differentiation
1 Tensors appear, p 36 - 38
2 Vector fields; the Riemann curvature tensor, p 38 - 42
3 Gauss’s Theorem Egregium: curvature is intrinsic, ie independent of the coordinate system, p 42 - 43
4 Geodesics and minimal surfaces; ex’s: the helicoid and catenoid again, p 44 - 47

5 Applications of geodesics: the exponential map, p 48 - 52

§IV MORE ADVANCED TOPICS

1 The Gauss-Bonnet theorem: surfaces with boundary, the Euler characteristic . . . , p 53 - 55

2 Generalization to higher dimensions, p 56 - 57

3 The Einstein-Hilbert equations, p 57 - 61

4 Cosmological constants, p 61 - 62

Final Homework: The Bianchi identities