

In lieu of a syllabus for 110.439 (differential geometry)

Differential geometry can be viewed as a kind of continuation of vector calculus (ie Calc III), to infinity and beyond – all the way down the throat of the Schwarzschild (black hole) solution of Einstein's equations for general relativity.

The table of contents below refers to a set of notes for the course, which is too big to post here. If you're interested, please contact me at jack@math.jhu.edu and I'll send you a copy, which will give you a better sense of the material covered in the course.

§I LOCAL VECTOR CALCULUS

- 1 Some Calc III and linear algebra, p 3 - 5
- 2 Curves and trajectories: Newton \Rightarrow Kepler, the tumbling asteroid, p 6 - 9
- 3 The moving (Frenet) frame, p 10 - 14 ; 15 - 16
- 4 Parametrized surfaces in \mathbb{R}^3 ; the implicit function theorem, p 18 - 20
- 5 Curves **on** surfaces; arc length and area; the 1st fundamental form, p 21 - 23

§II COORDINATE SYSTEMS AND THE TANGENT BUNDLE

- 1 Change of coordinates; the tangent space (see also p 13) and the chain rule, p 23 - 26
- 2 Digression on higher dimensions: the Schwarzschild metric, p 27 - 29
- 3 The 2nd fundamental form; ex's: the helicoid and torus, p 29 - 32
- 4 Eigenvectors and principal directions; ex: the catenoid, p 33 - 36

§III COVARIANT DIFFERENTIATION

- 1 Tensors appear, p 36 - 38
- 2 Vector fields; the Riemann curvature tensor, p 38 - 42
- 3 Gauss's Theorem Egregium: curvature is **intrinsic**, ie independent of the coordinate system, p 42 - 43

4 Geodesics and minimal surfaces; ex's: the helicoid and catenoid again, p 44 - 47

5 Applications of geodesics: the exponential map, p 48 - 52

§IV MORE ADVANCED TOPICS

1 The Gauss-Bonnet theorem: surfaces with boundary, the Euler characteristic . . . , p 53 - 55

2 Generalization to higher dimensions, p 56 - 57

3 The Einstein-Hilbert equations, p 57 - 61

4 Cosmological constants, p 61 - 62

Final Homework: The Bianchi identities