

# ALGEBRA EXAM

## Topics

### **Linear Algebra**

Abstract vector spaces; subspaces; dimension; inner products and orthogonal bases; matrices and linear transformations; matrix algebras and groups; determinants and traces; eigenvectors and eigenvalues, canonical forms; reduction of quadratic forms; bilinear forms; dual spaces; tensor products and tensor algebras.

### **Group Theory**

Groups and homomorphisms, Sylow, Jordan-Holder and Krull-Schmidt theorem; representations of finite groups; permutation representations; nilpotent, solvable and simple groups; matrix groups; free groups; extensions and group cohomology.

### **Commutative Rings**

Factorization and localization, chain conditions, Hilbert basis theorem, integral extensions, primary decompositions, Hilbert Nullstellensatz.

### **Module Theory**

Free and projective modules; tensor products; irreducible modules and Schurs lemma; semisimple, simple and primitive rings; density and Wederburn theorems; the structure of finitely generated modules over principal ideal domains, with application to abelian groups and canonical forms; categories and functors; complexes, cohomology; Tor and Ext.

### **Field Theory**

Field extensions, algebraic extensions, transcendence bases; cyclic and cyclotomic extensions; solvability of polynomial equations; finite fields; separable and inseparable extensions; Galois theory.

### **References**

Hungerford, Algebra  
Lang, Algebra  
Jacobson, Basic Algebra I  
Rotman, The theory of Groups