Existence of blow-up solutions in KdV-type equations

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Abstract: While the KdV equation and its generalizations with higher power nonlinearities (gKdV) have been long studied, a question about existence of blow-up solutions in the critical and supercritical cases has posed lots of challenges and far from being answered. One of the main obstacles is that unlike other dispersive models such as the nonlinear Schrodinger or wave equations, the gKdV equation does not have a suitable virial quantity, which is the key for showing existence of the finite time blow-up in other models. Only at the dawn of this century the groundbreaking works of Martel and Merle showed the existence of finite-time blow-up solutions in the critical gKdV equation.

We consider a higher dimensional extension of the gKdV equation, called generalized Zakharov-Kuznetsov (gZK) equation (the gKdV is limited as a spatially one-dimensional model), and ask if blow-up solutions exist in the corresponding critical gZK equation. We positively answer this question for the two-dimensional critical Zakharov-Kuznetsov equation. The new main ingredients include the Liouville-type theorem, pointwise decay estimates on the two dimensional Airy function, a la virial type quantities and understanding corresponding spectral properties. This is a joint work with Luiz Farah, Justin Holmer and Kai Yang.