Modeling Biological Fitness

Section Workshop 7- Project 4

Names of project team members:

Assign Roles for Project Teams

Roles will be assigned based on your birthdays. Whoever has the earliest birthday in the year is assigned to role 1, next birthday is role 2 and so on. If there are only 3 members in your group, combine roles 3 and 4.

1. Discussion coordinator - ensures all are heard
2. Notes coordinator - ensures documentation in a shareable form
3. Reporter - ready to speak for the group in with your Section Workshop leader and/or other groups
4. Submission coordinator - makes sure everyone understands what is left to do (if anything!) before the submission deadline, makes plans to get together, makes plans for uploading assignments to Gradescope (this person does not necessarily need to upload but is responsible for the group coming to a consensus on a plan).

Team project

Learning Goals

- In this project you will apply your understanding of graphing function, concavity, inflection points, absolute extreme to experiment with a model for biological fitness of organisms.
- You will work through optimization problems and be asked to write your own optimization problem.

Project work

You will work together through this content using Desmos to graph functions and investigate their behavior. You will connect this content to the problem of modeling the probability of survival of an organism.

Biological Fitness

What do you think of when you hear the term “fitness”? In day-to-day conversation “fitness” often refers to being “in-shape” and can bring to mind attributes like strength and endurance. In evolutionary biology, fitness has a completely different meaning that depends on many more factors than strength and endurance. Specifically, “relative fitness” refers to reproductive success of an organism; that is if the organism produces N offspring, the average number that will survive is called relative fitness. Relative fitness depends on many variables and how reproductive success is achieved varies greatly.

Strategies unrelated to what we think of as fitness (like mimicry, elaborate mating displays, establishing territories, sneak fertilization, etc.) can increase reproductive relative fitness. We will focus the discussion today on some basic variables that impact all organisms, including the age at which an organism first reproduces, the age at which it stops reproducing, the number of offspring that the organism has each year and how much of its resources it invests in raising each of them. Suppose that the total amount of resources that the organism can provide for its offspring is R and it has number of offspring N . R varies between organisms and also depends on the organism's environment. The constant number of resources invested in each offspring is usually defined by $x = \frac{R}{N}$. Let $f(x)$represent the probability of survival of offspring. A typical model for $f(x)$ is given by $f(x) = \frac{x^2}{x^2+k^2}$, where $k \geq 0$ is a constant. Then fitness is typically given by

$$w(x) = (\text{number of offspring}) \cdot (\text{probability of survival of offspring}) .$$

Group Report Questions

1) Use Desmos to graph $f(x)$ and add a slider for the constant k . Choose a value k and give a sketch of the graph for your choice of k . This kind of curve is called sigmoidal. What does the shape of the graph mean about the probability of survival? Consider
    - the roots of the function
    - inflection points
    - limit as x tends to 0 from the right
    - limit as x tends to ∞
    - concavity
2) With your group, write a paragraph explaining what the shape of the graph means about the probability of survival of an organism.
3) From the organism's point of view, the value of x that maximizes $w(x)$ is optimal.
    - Write the function $w(x)$ in terms of x and the constants k and R .
    - Find the value of x that maximizes $w(x)$. Your answer should be in terms of k and R .
4) Use Desmos to graph $w(x)$ and add sliders for k and R . Give a sketch of the graph for your choices of k and R . Does your conclusion from part (2) agree with the graph of the function? Use full sentences to explain your answer

Individual Project for Final Portfolio

Write your own optimization problem! You should choose a context for the problems that has meaning for you; think of an application that has come up in your daily life or is inspired by something you have learned in another class.

1. Write a paragraph explaining the context and its meaning for you. Be sure to include any citations for references that you use in your work.
2. Clearly write up the problem so that it can be handed to another student. Note: Please don't hand a fully written solution to your partner! Please exchange problems and work on your partner's problem.
3. Write up a complete solution to the problem, including diagrams. Fully explain your work.

Further Reading

Want to read more about biological fitness? This project was inspired by the Scitable by Nature blog.

Emlen, S. T. & Oring, L. W. Ecology, sexual selection, and the evolution of mating systems. Science 197, 215-223 (1977).

Gibson, K. N. Male mating tactics in spider monkeys: Sneaking to compete. American Journal of Primatology 72, 794-804 (2010).

Hoffman, C. et al. Sex differences in survival costs of reproduction in a promiscuous primate. Behavioral Ecology and Sociobiology 62, 1711-1718 (2008).

Kappeler, P. M. "Mate choice," in Evolution of Primate Societies, eds. J. C. Mitani et al. (Chicago, IL: University of Chicago Press, 2012) 367-386.

Kappeler, P. M. (2012) Male Reproductive Strategies. Nature Education Knowledge 3(10):82

This assignment was prepared by Emily Braley, Johns Hopkins University Mathematics, Spring 2022.