# Proposal: A writing intensive version of

# AS.110.405 Real Analysis I

*By Richard Brown*

*Abstract:* With the creation and expansion of the University Writing Program (UWP), along with the goals and recommendations of the Second Commission on Undergraduate Education (CUE2) here at Hopkins, defining what it means to write within disciplines like mathematics and other STEM majors has become a strong focus within the Krieger School. The importance of mathematical writing in the learning process, writing in different formats, for different learning goals, and for different audiences within (and outside of) the mathematical community, is well-defined in the field (see Knuth, et.al [1], Sterret, [2], and Meier and Rishel [3]), and is a fundamental part of the Mathematical Association of America's (MAA's) Instructional Practice Guide [4]). However, writing in mathematics, either for simple communication or to augment the learning process, is hardly ever explicitly taught at the undergraduate level. Undergraduate students are simply expected to learn how to write mathematics through the learning process of doing mathematics. To correct this lack of a dedicated and structured focus on writing within a mathematics curriculum and teach the writing of mathematics alongside and within the learning of the math, we propose to redesign one of our core required courses for the mathematics major **AS.110.405 Real Analysis I**, as a UWP-approved writing intensive course using the above references as a guide. This redesign would include enhancing the curriculum to incorporate a diverse set of high and low stakes assessed writing assignments to teach the effective written communication of mathematics along with the actual understanding of the mathematics. We will work closely with the UWP in the design of both assignments and assessments strategies, as well as the redesign of the lecture and section contact hours to focus on both content and writing style within the instruction. The UWP will help train and brief mathematics instructional staff on best practices in the teaching, mentorship, and assessment of good writing, and act as mentors throughout the implementation of the course. The goal is to augment the curriculum of **AS.110.405** without sacrificing any of the mathematical content learning the course already incorporates.

## PI Biography

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Bio: Richard Brown is a Teaching Professor and Director of Undergraduate Studies in Mathematics here at Johns Hopkins University. He has been an active member of the JHU Teaching Academy since its inception, and is a current KSAS Senator for the Natural Sciences and a Vice Chair of its Steering Committee. He has been teaching university-level mathematics since the early 90's and has been mentoring graduate students and early-stage postdocs in the art of teaching since 2005 here at Hopkins. He has also recently incorporated portfolio and project-based activities in the large-lecture course **AS.110.302 Differential Equations**, geared toward different audiences, with presentations and peer-reviews among students. And he is currently implementing a project-based approach to the learning of single variable calculus to 140 students in the course AS.110.107 Calculus II (Bio & Soc Sci).

## Project Description

The course **AS.110.405 Real Analysis I** is a regular part of the mathematics curriculum in the Mathematics Department here at Hopkins. It is a core requirement for the mathematics major and is a course offered each semester of the academic year. It is a course with an average enrollment of 21 students over its last 5 instances. Many mathematics majors who are interested in graduate school in pure mathematics would opt for the honors version of the course, **AS.110.415 Honors Analysis I** instead of **AS.110.405**. This non-honors version is usually taken by mathematics majors who also have a second major and/or simply want a solid foundation in real analysis along with their other interests. It can be considered a ***first-proof-based course*** for students, and the course is often taken by students without any prior training in how to write a mathematical proof. Training students to write mathematical proofs is not an explicit part of the instructional curriculum, but developing the skills of writing a proof is a feature of the course as a consequence of the presentation and assessment of the accuracy of problem set submissions and exam solutions.

The course description can be summarized as:

> ***AS.110.405 Real Analysis I*** – A course in the formal development of the properties of the real numbers and the structures defined on them, including set theory, sequences and series, functions, and the continuity, differentiability and integrability of functions.

We propose to redesign the curriculum of this course from the traditional lecture-section-homework-exam structure to a writing-intensive one, using the current implementation of such by the UWP without sacrificing the content and its use as a prerequisite for many other courses within the major. The UWP's current criteria for a "W" course includes: (1) Multiple substantial writing projects, (2) a mix of high and low stakes writing, (3) direct instruction on writing, (4) assignments with clearly conveyed expectations, including the genre and audience of the assigned writing, and evaluative criteria, such as rubrics, (5) feedback to students on their writing, and (6) at least one opportunity to revise. These criteria align well with strategies detailed in the above-mentioned references on how to incorporate writing into the curriculum to enhance learning in mathematics.

We envision a course in the following way:

- Full-coverage lectures on the topics of the content syllabus for AS.110.405 Real Analysis I would be augmented by:
    - More focus on the form of mathematics presentation within the lecture environment, presenting this form early on as part of the content instruction, and reinforcing this focus throughout the course.
    - Highlighting of stylistic elements involved in mathematical presentation, proof-writing, and concept development.
        - Through structured worksheets and handouts, websites, etc.
        - Reinforced in lecture through stylized and accentuated writing exposure in context.
    - Small group activities like definition writing, direct vs. reverse engineered proof-writing, example and counterexample development brainstorming, etc.
        - In lecture class small groups activities,

        - TA section meeting activities.
- TA section meetings would be less Q/A, and redesigned (However, among the bullets below, the content inside the focus on style remains the actual mathematics that they are learning):
    - More of a focus on writing style and form in various ways.
    - Small presentations with formative feedback of writing samples for future revision and submission, along with peer critique to be handed back to the presenter.
    - Workshops in how to write clearly and effectively for various audiences and in different forums.
- Extra TA/CA Office hours devoted entirely to writing.
- No timed written examinations during the course and a short, timed final exam, for course assessment purposes.

The writing elements of the course will include:

- **Journal:** Course end peer-reviewed fake Analysis journal submission at the end of the course.
    - Content of these submissions would be items relevant to the course but on subjects adjacent to core course material. Things like Taylor's Theorem, Picard Iterations, the decimal representation of the reals, and other standalone topics of interest to all, but would require a bit of extra research outside of the course.
    - A list of possible topics to choose from will be provided, but other topics are encouraged as approved by the instructional staff.
    - Submissions would be peer-reviewed by other students in the class, with the idea of a peer-review also taught and treated as a writing assignment for grade.
    - Each student would construct their own article, and peer-review two others.
- **Presentation:** The journal submission at the end of the course will include a small presentation to the class, allowing the author to practice the transition from formal writing to formal presentation of mathematical material. (The writing of slides for such a presentation would be taught and assessed as part of the focus.)
- **Portfolio:** A set of artifacts, offered for submission throughout the semester on a regular basis.
    - Based on re-worked and polished homework problems that have already been graded and critiqued by the TA,
    - Geared toward different audiences (e.g., peers, professor, textbook style, etc.).
    - As part of the TA section meetings, students cycle through short presentations of some recent portfolio submission. The TA leads a discussion of the work within the class. Subsequent mini peer reviews of small presentations can be either written or oral.
- **Diary:** A set of small low and no stakes assignments to write to oneself on elements of the learning process and to document growth in understanding.
    - In here would go reflections, restatements, connections, and other assessments of one's own learning process.
    - Periodically submitted for review by instructional staff.
- **Problem Sets:** The backbone of a mathematics course, proof-writing for understanding is a weekly endeavor, submitted and graded.
    - These would still exist, and one problem each week would eventually feed the portfolios.
    - Individual problems may involve audience-specific focus.
    - Most graded for accuracy, some graded also for style, with commentary.
    - Revisions of those graded for accuracy and style would wind up being portfolio artifacts.
- **A short, in-class timed final exam, mostly for course assessment purposes**.

## Proposed resource needs and budget:

- A GTA/UTA from the regular pool of the Mathematics Department who volunteers to have this as an assignment.
- 2 CAs, hired separately for this project alone, to begin work over the summer and throughout the semester as active instructional staff.

## Budget

- PI: $1500 for project. $1500
- August training period:
  - TA: 30 hours at $15/hr. $450
  - 2CAs: 30 hours at $15/hr. $900
- Fall semester:
  - 1/2 buy out of TA duties. (replace with a CA elsewhere) $1500
  - 2CAs: 100 hours at $15/hr $3000
  - Focus groups and course evaluation
    - Selected focus group participants JHU B&N gift cards $400
    - Recorded audio transcribed for qualitative analysis $400
- Total: $8150.

## Time-line for the development of the proposed course:

- Summer 2023: Pre-course development
  - Meet with assigned TA and hire CAs to create an instructional staff.
  - Meet as a team with UWP staff to begin the training on writing assignment critique and assessment, and rubric construction.
  - Develop the course structure to implement assessment types, assignment strategies, and course tools.
    - Build out weekly curriculum and lectures,
    - Construct specific course materials, including worksheets, activities, and workshops.
    - Develop materials for LaTeX instruction.
  - Build out the Canvas course to serve as the central repository for course engagement.
  - Develop the instructional team's complementary roles for the instructor/TA/CAs.
- Fall 2023: Run the course
  - Hold weekly meetings with course instructional staff to continually assess workload, student engagement, and focus.
  - Tweak as necessary to uphold course aims.
  - Collect and assess student feedback on a regular basis for analysis.
- Spring 2024: Post-course analysis.
  - Follow students in the successive course in the sequence.
  - Package all materials and resources for distribution to future use.
  - Perform qualitative analysis on focus group data.
  - Write-up a post-course report on the experience, including student reactions.
  - Report to the community on the results, including presenting at the JMM, and also in house to the Hopkins community.

## Evaluation and Assessment

To evaluate the effectiveness of the redesigned curriculum, we will employ a number of tools, both within the course itself, and outside of the course:

1. We will offer a short in-class final exam in a traditional manner during the final exam slot, for use to compare against the final exam of a previous version of the course, taught in the Fall 2022 semester by the PI using the same text and content syllabus.
2. We will survey all students in the course at three intervals of the course: at the 4$^{th}$ week, the 8$^{th}$ week, and the end, to gauge understanding, workload, course structure, pace, and satisfaction.
3. We have retained and documented samples of homework assignments from the Fall 2022 version of the course and will assign similar problems along the way in this course to compare directly student learning through homework submission. This will become a direct comparison of mathematical competency as well as writing style between this course and last year's.
4. We will gather qualitative data through focus group discussions to learn more about student experiences in the course. We want to learn how students interact with various writing components in the course through their recorded impressions. The results of this study will help inform the organization and structure of future offerings of the writing intensive version of this course.
5. We will follow the progress of students in the second course of the same sequence of courses **AS.110.406 Real Analysis II**.

## Bibliography


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